Mandatory versus Discretionary Spending:
The Status Quo Effect†

By T. Renee Bowen, Ying Chen, and Hülya Eraslan*

Do mandatory spending programs such as Medicare improve efficiency? We analyze a model with two parties allocating a fixed budget to a public good and private transfers each period over an infinite horizon. We compare two institutions that differ in whether public good spending is discretionary or mandatory. We model mandatory spending as an endogenous status quo since it is enacted by law and remains in effect until changed. Mandatory programs result in higher public good spending; furthermore, they ex ante Pareto dominate discretionary programs when parties are patient, persistence of power is low, and polarization is low. (JEL C78, E62, H41, H61)

Government budgets are primarily decided through negotiations. Institutions governing budget negotiations play an important role in fiscal policy outcomes. These institutions vary across countries and time, and examining their effects is an important step towards understanding these variations. In this paper, we are interested in the role of a particular institution: mandatory spending programs.

Mandatory spending is expenditure that is governed by formulas or criteria set forth in enacted law, rather than by periodic appropriations. As such, unless explicitly changed, the previous year’s spending bill applies to the current year. By contrast, discretionary spending is expenditure that is governed by annual or other periodic appropriations. Examples of mandatory spending in the United States include entitlement programs such as Social Security and Medicare, while discretionary spending consists of mostly military spending. As Figure 1 shows, mandatory spending in the United States has been growing as a share of gross domestic product (GDP). In 2012, mandatory spending was $2 trillion compared to discretionary spending of $1.3 trillion, attributed mostly to entitlement programs. Because of these trends, entitlement programs have been at the heart of recent budget negotiations and are

† Go to http://dx.doi.org/10.1257/aer.104.10.2941 to visit the article page for additional materials and author disclosure statement(s).
consistently ranked as a top issue by the public and policymakers. There has been substantial research on entitlement programs, but these studies have focused on dimensions of these programs other than their mandatory nature—for example, indexation (Boskin and Jorgenson 1997), funding mechanisms, and transparency (Feldstein 2005). Little research has been done to explore the difference in mandatory versus discretionary spending in budget negotiations.

We take a first step towards understanding the effects of mandatory spending programs on budget negotiations and their implications for the efficient provision of public goods. In our model, two parties decide how to allocate an exogenously given budget to spending on a public good and private transfers for each party in every period over an infinite horizon. Parties potentially differ in the value they attach to the public good and we refer to the degree of such differences as the level of polarization between the parties. Each period a party is randomly selected to make a budget proposal. The probability that the last period’s proposer is selected

---

2 See http://www.people-press.org/2012/06/14/debt-and-deficit-a-public-opinion-dilemma/

3 There is a growing dynamic political economy literature beginning with Epple and Riordan (1987) that models the status quo policy as the policy in place in the previous period. This literature is motivated by the observation that most laws and government programs are continuing and remain in effect in the absence of new legislation. We are the first to explicitly compare mandatory programs to discretionary programs. We discuss these papers in the related literature section.

4 The definition of a public good requires it to be nonexcludable and nonrivalrous in consumption. However, our model only requires that the good be nonexcludable, and as such, is also applicable to a common pool resource. Entitlement programs such as Social Security and Medicare are often thought of as a common pool resource.
to be the proposer in the current period captures the *persistence of political power*. The proposer makes a take-it-or-leave-it budget offer. If the other party accepts the offer, it is implemented; otherwise, the status quo prevails. We compare two institutions that govern the status quo: a political system in which public good spending is discretionary, in which case the status quo public good allocation is set to zero each period; and a political system in which public good spending is mandatory, in which case the status quo public good allocation is what was implemented in the previous period, and hence is endogenous. Under both institutions, we assume that the status quo allocation to private transfers is zero.

Under discretionary public spending, in the unique Markov perfect equilibrium, the party in power underprovides the public good and extracts the maximum private transfer for itself. This is because there is no dynamic link between policy chosen today and future outcomes with discretionary programs. Hence the optimal choice of public good for the proposer is its static optimal choice, which is below the efficient level, and the proposer is able to implement this because discretionary programs give the responding party no bargaining power. Under discretionary programs the steady-state distribution of public good spending follows a Markov process governed by the persistence of power: the level of the public good changes only when the proposing party changes.

Under mandatory public spending, the degree of polarization plays an important role. We characterize Markov perfect equilibria first when polarization is low and second when polarization is high.

In the low-polarization case, the levels of public good spending proposed by both parties are either below or equal to the efficient level in both transient and steady states, and are always closer to the efficient level than when public good spending is discretionary. To understand why, note that mandatory programs create a channel to provide insurance against power fluctuations because they raise the bargaining power of the nonproposing party by raising its status quo payoff. When the status quo level of the public good is very low, the party that places a higher value on the public good (party $H$) exploits the weak bargaining position of the party that places a lower value on the public good (party $L$), and proposes its dynamic ideal. Because of the insurance motive, party $H$’s dynamic ideal is strictly above its static ideal (the level it would propose with discretionary programs). Indeed, the set of steady-state levels of the public good in the low-polarization case is a continuum from party $H$’s dynamic ideal to the efficient level. In the high-polarization case, the insurance effect from mandatory programs can lead party $H$ to propose a level of public good spending above the efficient level, creating temporary overprovision. This is only temporary because of power fluctuations—once party $L$ comes to power, it lowers the level of public good to the efficient level and provides transfers to party $H$ so that it accepts. Anticipation of the transfers gives party $H$ the incentive to overprovide the public good. The unique steady-state level of public good spending in the high-polarization case is the efficient level.

As is typical in dynamic games, we cannot appeal to general theorems on uniqueness of Markov perfect equilibrium, but we show that under some conditions, there are no steady states other than the ones in the equilibria we characterize in the game with mandatory public spending. This allows us to conduct comparative statics and make welfare comparisons.
One interesting comparative static is that greater power fluctuations (lower persistence of power) improve efficiency with mandatory programs. This is because greater power fluctuations provide stronger insurance incentives, leading to a higher steady-state level of public good. This is in contrast to Besley and Coate (1998), which shows that power fluctuations undermine policymakers’ incentives to invest in public goods, leading to less efficient outcomes.

Perhaps it is not surprising that party \( H \) benefits from mandatory programs. But strikingly, party \( L \) also benefits from mandatory programs, provided that the parties are patient, the persistence of power is low, and polarization is low. Intuitively, if party \( L \) cares sufficiently about future payoffs, expects power to fluctuate frequently, and the value it places on the public good is not too low, then the insurance benefit from mandatory programs is high, making party \( L \) better off. Thus, mandatory programs can be Pareto improving, and this may explain why they are successfully enacted in the first place.

To check the robustness of our results, we also examine the case of all spending mandatory in which the status quo of both the public good spending and private transfers is the previous period’s allocation. We find in this case that steady states are Pareto efficient whereas in the case of all discretionary, the allocation of the public good is not Pareto efficient. This provides further evidence that there is an advantage to mandatory programs.

**Related Literature.**—The distinction between private goods and public goods goes back to at least Smith (1776), who concluded that public goods must be provided by the government since the market fails to do so. By now there exists a vast literature formally studying public goods, starting with the classic work by Wicksell (1896) and Lindahl (1919).

Our paper adds to the literature on public goods provision with political economy frictions as surveyed in Persson and Tabellini (2000). A subset of this literature analyzes public good provision under different political institutions. For example, Lizzieri and Persico (2001) compare the provision of public goods under different electoral systems. The particular institution that our paper focuses on is mandatory spending programs.

We consider public good provision in a legislative bargaining framework, similar to Baron (1996); Leblanc, Snyder, and Tripathi (2000); Volden and Wiseman (2007); and Battaglini and Coate (2007, 2008). With the exception of Baron (1996), these papers do not consider mandatory programs. Baron (1996) presents a dynamic theory of bargaining over public goods programs in a majority-rule legislature where the status quo in a session is given by the program last enacted. He models the provision of public goods as a unidimensional policy choice, and analyzes the equilibrium outcome under mandatory programs only. Our paper contributes to this literature by analyzing a multidimensional policy choice involving both mandatory and discretionary programs and exploring the efficiency implications.

Building on the seminal papers of Rubenstein (1982) and Baron and Ferejohn (1989), most papers on political bargaining study environments where the game ends once an agreement is reached. Starting with the works of Eppe and Riordan (1987) and Baron (1996), there is now an active literature on bargaining with an endogenous status quo. This literature includes Baron and Herron (2003); Kalandrakis (2004);...
Bernheim, Rangel, and Rayo (2006); Anesi (2010); Bowen (2014); Diermeier and Fong (2011); Zápal (2012); Anesi and Seidmann (2012); Bowen and Zahran (2012); Duggan and Kalandrakis (2012); Dziuda and Loeper (2013); Nunnari (2012); Piguillem and Riboni (2012); and Baron and Bowen (2013). These papers consider bargaining over either a unidimensional policy or the division of private benefits. Thus, they do not address how mandatory programs affect the provision of public goods in budget negotiations, which is at the heart of our paper.

Our work is also related to the literature on power fluctuations, which includes Persson and Svensson (1989); Alesina and Tabellini (1990); Besley and Coate (1998); Grossman and Helpman (1998); Hassler, Storesletten, and Zilibotti (2007); Klein, Krusell, and Ríos-Rull (2008); Azzimonti (2011); and Song, Storesletten, and Zilibotti (2012). These papers show that power fluctuations can lead to economic inefficiency. We show that this inefficiency may be attenuated by mandatory spending programs. By considering equilibria that are non-Markov, Dixit, Grossman, and Gül (2000) and Acemoglu, Golosov, and Tsyvinski (2011) establish the possibility of political compromise to share risk under power fluctuations. Our paper shows, in contrast, that even if parties use Markov strategies, they can reach a certain degree of compromise with mandatory programs because the party in power cannot fully undo the decisions of the past. Moreover, we discuss political compromise in the context of public good provision, which has efficiency implications beyond risk sharing.

Mandatory programs generate a dynamic link between policy in a given period and political power in future periods. In that sense, our paper is also related to Bai and Lagunoff (2011), who analyze policy endogenous power.

In the next section we describe our model. In Section II we characterize Pareto efficient allocations. In Section III we define a Markov perfect equilibrium for our model. We analyze discretionary public spending in Section IV and mandatory public spending in Section V. We discuss efficiency implications of mandatory public spending in Section VI, and analyze the case of all mandatory programs in Section VII. In Section VIII, we conclude and discuss some important extensions.

I. Model

Consider a stylized economy and political system with two parties labeled $H$ and $L$. Time is discrete, infinite, and indexed by $t$. Each period the two parties decide how to allocate an exogenously given dollar. The budget consists of an allocation to spending on a public good, $g^t$, and private transfers for each party, $x^t_H$ and $x^t_L$. Denote by $b^t = (g^t, x^t_H, x^t_L)$ the budget implemented at time $t$. Let $B = \{ y \in \mathbb{R}_+^3 : \sum_{i=1}^3 y_i \leq 1 \}$. Feasibility requires that $b^t \in B$. The stage utility for party $i$ from the budget $b^t$ is

$$u_i(b^t) = x^t_i + \theta_i \ln(g^t),$$

where $\theta_i$ is the weight of the public good relative to the transfer for party $i \in \{H, L\}$.\(^5\)

We assume $\theta_H \geq \theta_L > 0$ and $\theta_H + \theta_L < 1$. The latter condition ensures that the

\(^5\)We assume log utility for tractability. This functional form is commonly used in economic applications. See, for example, Azzimonti (2011) and Song, Storesletten, and Zilibotti (2012). The results are qualitatively the same in the numerical analysis using constant relative risk aversion (CRRA) utility functions.
efficient level of public good spending is lower than the size of the budget, as we show later in Section II.

The parties have a common discount factor \( \delta \in (0, 1) \). Party \( i \) seeks to maximize its discounted dynamic payoff from an infinite sequence of budgets, \( \sum_{t=0}^{\infty} \delta^t u_i(b') \).

**Political System.**—We consider a political system with unanimity rule. Each period a party is randomly selected to make a proposal for the allocation of the dollar. The probability of being the proposer is Markovian. Specifically, \( p \) is the probability that party \( i \) is the proposer in period \( t + 1 \) if it was the proposer in period \( t \). We interpret \( p \) as the persistence of political power.

At the beginning of period \( t \), the identity of the proposing party is realized. The proposing party makes a proposal for the budget, denoted by \( z' \). If the responding party agrees to the proposal, it becomes the implemented budget for the period, so \( b' = z' \); otherwise, \( b' = s' \), where \( s' \) is the status quo budget.

Let \( S \subseteq B \) be the set of feasible status quo budgets, and let \( \zeta : B \rightarrow S \) be a function that maps the budget in period \( t \) to the status quo in period \( t + 1 \). So \( s^{t+1} = \zeta(b') \) for all \( t \). The set \( S \) and the function \( \zeta \) are determined by the rules governing mandatory and discretionary programs. For example, if no mandatory programs are allowed, then \( S = \{(0, 0, 0)\} \) and \( \zeta(b) = (0, 0, 0) \) for all \( b \in B \). That is, in the event that the proposal is rejected, no spending occurs that period. At the other extreme where all spending is in the form of mandatory programs, \( S = B \) and \( \zeta(b) = b \); that is, disagreement on a new budget implies the last period’s budget is implemented.

We focus on comparing two institutions: one in which all spending is discretionary (that is, \( \zeta(b) = (0, 0, 0) \)), and the other in which spending on the public good is mandatory, but private transfers are discretionary (that is, \( \zeta(b) = (g, 0, 0) \) for any \( b = (g, x_H, x_L) \)). We find it reasonable to think of the US federal budget as allocating private transfers through discretionary spending and public goods through mandatory programs. This is because transfers designated for particular districts (for example, earmarks) are typically appropriated annually, whereas social programs such as Social Security and Medicare are funded through mandatory programs and provide benefits from which constituents of any particular party cannot be excluded. As mentioned in the introduction, although Social Security and Medicare do not satisfy the “nonrivalrous” criterion, they satisfy the “nonexcludable” criterion and are therefore often thought of as a common pool resource. Our model applies when \( g \) is a common pool resource; for expositional convenience, we refer to \( g \) as a “public good.”

---

6 Most political systems are not formally characterized by unanimity rule; however, many have institutions that limit a single party’s power, for example, the “checks and balances” included in the US Constitution. Under these institutions, if the majority party’s power is not sufficiently high, then it needs approval of the other party to set new policies.

7 While we think the case of public good mandatory and private transfers discretionary is salient, there are other interesting status quo rules to consider. For example, a form of transfer may be returning a portion of the budget to constituents in each party’s electoral coalition through tax reductions. Since transfers through the tax code are usually automatically renewed, in this interpretation transfers are mandatory. For robustness we analyze the case of all mandatory spending in Section VII. We find qualitatively similar results.

8 Our results would go through if instead we assumed \( u_i(b') = x'_i + \theta_i \ln(a_i g') \) for some constant \( a_i > 0 \). We can think of \( a_i/(a_H + a_L) \) as the fraction of the common pool resource party \( i \) extracts in a second stage game after the total allocation to the public good is agreed upon. In that sense, our results apply to settings where \( g' \) is nonexcludable but not necessarily nonrivalrous.
II. Pareto Efficient Allocations

As a benchmark, consider the Pareto efficient allocations. A Pareto efficient allocation solves the following problem for some $\_U \in \mathbb{R}$:

$$\max_{\{b^t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t u_L(b^t)$$

s.t. $\sum_{t=0}^{\infty} \delta^t u_H(b^t) \geq \_U$ and $b^t \in B$ for all $t$. 

We find that any Pareto efficient allocation with $x^t_L > 0$ and $x^{t''}_L > 0$ for some $t'$ and $t''$ must have $g^t = \theta_H + \theta_L$ for all $t$. Note also that $g^t = \theta_H + \theta_L$ is the unique Samuelson level of the public good. We henceforth refer to $\theta_H + \theta_L$ as the efficient level of the public good.

For contrast, consider party $i$’s ideal allocation in any period, which solves $\max_{b \in B} u_i(b)$. Let us call the level of public good that solves this problem the dictator level for party $i$. Clearly party $i$ would not choose to allocate any spending to party $j$, hence the dictator level solves $\max_g 1 - g + \theta_i \ln(g)$. This is maximized at $\theta_i < \theta_H + \theta_L$. So party $i$’s ideal level of the public good results in underprovision of the public good.

III. Markov Perfect Equilibrium

We consider stationary Markov perfect equilibria. A Markov strategy depends only on payoff-relevant events, and a stationary Markov strategy does not depend on calendar time. In our model, the payoff-relevant state in any period is the status quo $s$. Thus, a (pure) stationary Markov strategy for party $i$ is a pair of functions $\sigma^i = (\pi^i, \alpha^i)$, where $\pi^i : S \rightarrow B$ is a proposal strategy for party $i$ and $\alpha^i : S \times B \rightarrow \{0, 1\}$ is an acceptance strategy for party $i$. Party $i$’s proposal strategy $\pi^i = (\gamma^i, \chi^i_H, \chi^i_L)$ associates with each status quo $s$ an amount of public good spending, denoted by $\gamma^i(s)$, an amount of private transfer for party $H$, denoted by $\chi^i_H(s)$, and an amount of private transfer for party $L$, denoted by $\chi^i_L(s)$. Party $i$’s acceptance strategy $\alpha^i(s, z)$ takes the value 1 if party $i$ accepts the proposal $z$ offered by party $j \neq i$ when the status quo is $s$, and 0 otherwise. A stationary Markov perfect equilibrium is a subgame perfect Nash equilibrium in stationary Markov strategies. We henceforth refer to a stationary Markov perfect equilibrium simply as an equilibrium.

---

9 A proof is available in the online Appendix.

10 The Samuelson rule for the efficient provision of public goods requires that the sum of the marginal benefits of the public good equals its marginal cost.

11 This is a commonly used solution concept in dynamic political economy models. See, for example, Battaglini and Coate (2008); Diermeier and Fong (2011); and Dziuda and Loeper (2013). It is reasonable in dynamic political economy models where there is turnover within parties since stationary Markov equilibria are simple and do not require coordination. Similar to Dixit, Grossman, and Güll (2000) and Acemoglu, Golosov, and Tsyvinski (2011), there exist non-Markov equilibria with properties different from the Markov equilibrium we characterize, as we show in footnote 14.
To each strategy profile \( \sigma = (\sigma_H, \sigma_L) \), and each party \( i \), we can associate two functions: \( V_i(\cdot; \sigma) \) and \( W_i(\cdot; \sigma) \). The value \( V_i(s; \sigma) \) represents the dynamic payoff of party \( i \) if \( i \) is the proposer in the current period and the value \( W_i(s; \sigma) \) represents the dynamic payoff of party \( i \) if \( i \) is the responder in the current period, when the status quo is \( s \) and the strategy profile \( \sigma \) will be played from the current period onward.

We restrict attention to equilibria in which (i) \( \alpha'(s, z) = 1 \) when party \( i \) is indifferent between \( s \) and \( z \); and (ii) \( \alpha'(s, \pi'(s)) = 1 \) for all \( s \in \mathcal{S} \), \( i, j \in \{H, L\} \) with \( j \neq i \). That is, the responder accepts any proposal that it is indifferent between accepting and rejecting, and the equilibrium proposals are always accepted.\[\text{12}\] Given the restriction that equilibrium proposals are always accepted, in these equilibria the implemented budget is the proposed budget.

Call a strategy profile \( \sigma \) and associated payoff quadruple \( (V_H, W_H, V_L, W_L) \) a strategy-payoff pair. In what follows, we suppress the dependence of the payoff quadruple on \( \sigma \) for notational convenience. Given the restrictions that parties accept when indifferent and equilibrium proposals are always accepted, a strategy-payoff pair is an equilibrium strategy-payoff pair if and only if

(E1) Given \( (V_H, W_H, V_L, W_L) \), for any proposal \( z = (g', x'_H, x'_L) \in \mathcal{B} \) and status quo \( s = (g, x_H, x_L) \in \mathcal{S} \), the acceptance strategy \( \alpha'(s, z) = 1 \) if and only if

\[
(1) \quad x'_i + \theta_i \ln(g') + \delta[(1-p)\nu_i(\zeta(z)) + p\nu_i(\zeta(z))] \geq K_i(s),
\]

where \( K_i(s) = x_i + \theta_i \ln(g) + \delta[(1-p)\nu_i(s) + p\nu_i(s)] \) denotes the dynamic payoff of \( i \) from the status quo \( s = (g, x_H, x_L) \).

(E2) Given \( (V_H, W_H, V_L, W_L) \) and \( \alpha' \), for any status quo \( s = (g, x_H, x_L) \in \mathcal{S} \), the proposal strategy \( \pi'(s) \) of party \( i \neq j \) satisfies:

\[
(2) \quad \pi'(s) \in \arg \max_{z = (g', x'_H, x'_L) \in \mathcal{B}} x'_i + \theta_i \ln(g') + \delta[p\nu_i(\zeta(z)) + (1-p)\nu_j(\zeta(z))] \geq K_j(s).
\]

\[
(3) \quad x'_j + \theta_j \ln(g') + \delta[(1-p)\nu_j(\zeta(z)) + p\nu_j(\zeta(z))] \geq K_j(s).
\]

\[\text{12}\] Any equilibrium is payoff equivalent to some equilibrium (possibly itself) that satisfies (i) and (ii). We take two steps to show this: first, any equilibrium is payoff equivalent to some equilibrium that satisfies (i); second, any equilibrium that satisfies (i) is payoff equivalent to some equilibrium that satisfies (i) and (ii).

To prove the first step, consider an equilibrium \( \sigma^E \) that does not satisfy (i). Then there exists a status quo \( s' \) and a proposal \( z' = (g', x'_H, x'_L) \) such that the responder \( i \) is indifferent between \( s' \) and \( z' \) but \( \alpha'(s', z') = 0 \). If \( z' \) gives the proposer \( j \) a lower payoff than \( \pi'(s') \), then \( \sigma^E \) is payoff equivalent to the equilibrium which is the same as \( \sigma^E \) except that \( \alpha'(s', z') = 1 \) because \( j \) would not propose \( z' \) when the status quo is \( s' \). If \( z' \) gives the proposer a strictly higher payoff than \( \pi'(s') \), then there exists a proposal \( z'' \) that gives the responder a higher payoff than \( z' \) does and gives the proposer a strictly higher payoff than \( \pi'(s') \). That is, \( z'' \) is a strictly better proposal than \( \pi'(s') \), contradicting that \( \sigma^E \) is an equilibrium.

To prove the second step, consider an equilibrium \( \sigma^E \) that satisfies (i) but not (ii). Then there exists a status quo \( s' \) such that \( \alpha'(s', \pi'(s')) = 0 \), implying that the proposer receives the status quo payoff by proposing \( \pi'(s') \) when the status quo is \( s' \). By condition (i), the status quo is a proposal that is accepted. It follows that \( \sigma^E \) is payoff equivalent to the equilibrium which is the same as \( \sigma^E \) except that \( \pi'(s') = s' \).
(E3) Given \( \sigma = ((\pi^H, \alpha^H), (\pi^L, \alpha^L)) \), the payoff quadruple \((V_H, W_H, V_L, W_L)\) satisfies the following functional equations for any \( s = (g, x_H, x_L) \in S, i, j \in \{H, L\} \) with \( j \neq i \):

\[
V_i(s) = \chi^i(s) + \theta_i \ln(\gamma^i(s)) + \delta [pV_i(\zeta(\pi^i(s))) + (1 - p)W_i(\zeta(\pi^i(s)))] \\
W_i(s) = \chi^i(s) + \theta_i \ln(\gamma^j(s)) + \delta [(1 - p)V_i(\zeta(\pi^j(s))) + pW_i(\zeta(\pi^j(s)))]
\]

Condition (E1) says that the responder accepts a proposal if and only if its dynamic payoff from the proposal is higher than its status quo payoff. Condition (E2) requires that for any status quo \( s \), party \( i \)'s equilibrium proposal maximizes its dynamic payoff subject to party \( j \) accepting the proposal. Condition (E3) says that the equilibrium payoff functions must be generated by the equilibrium proposal strategies.

We begin by considering the benchmark model of all discretionary, and then consider the model in which spending on the public good is mandatory and private transfers are discretionary.

### IV. Discretionary Public Spending

Suppose all spending is discretionary, implying that the status quo level of public good spending as well as private transfers is zero. That is, \( \zeta(b) = (0, 0, 0) \) for any \( b \in B \).\(^{13}\) Because of log utility in the public good, the responder’s status quo payoff \( K_i(s) \) is \(-\infty\) for any status quo \( s \), and hence the responder’s acceptance constraint is not binding. The proposer therefore sets the public good at the dictator level \( \theta_i \) every period and there is underprovision of the public good. This leads to the following proposition.\(^{14}\)

**PROPOSITION 1:** If all spending is discretionary, then the public good is provided at the dictator level, and there is underprovision of the public good in the unique equilibrium.

One implication of Proposition 1 is that with only discretionary spending, the equilibrium allocation to the public good follows a Markov process. Specifically, if \( i \) is the proposer in the current period, spending on the public good next period is \( \theta_i \) with probability \( p \) (if \( i \) is the proposer in the next period), and \( \theta_j \) with probability \( 1 - p \) (if \( j \) is the proposer in the next period). Note that with only discretionary spending, the equilibrium allocation is dynamically Pareto inefficient since both parties receive private transfers in some periods, but the spending on the public good is below \( \theta_H + \theta_L \) in every period.\(^{15}\) In Section VB, we compare this long-run behavior...
of spending on the public good under discretionary programs to the long-run behavior under mandatory programs, and assess the efficiency implications in Section VI.

V. Mandatory Public Spending

We now consider the case in which only the public good spending is mandatory; that is, $\zeta(b) = (g, 0, 0)$ for any $b = (g, x_H, x_L) \in \mathcal{B}$. In the rest of this section to lighten notation we write $\pi^i(g)$, $\alpha^i(g, z)$, $V_i(g)$, $W_i(g)$, and $K_i(g)$ instead of $\pi^i(s)$, $\alpha^i(s, z)$, $V_i(s)$, $W_i(s)$, and $K_i(s)$. We also refer to the status quo public good level as the status quo.

We show in this section that if public good spending is mandatory, then the public good is provided at a higher level than under discretionary spending. The reason is that mandatory spending improves the status quo payoff of the responding party, and hence increases its bargaining power.\footnote{This result does not depend on only the public good being provided under mandatory programs. We discuss the case in which all spending is allocated through mandatory programs in Section VII.}

To obtain some intuition, we first analyze a one-period model with an exogenous status quo and then analyze the infinite-horizon game.

A. A One-Period Model

Suppose party $i$ is the proposer and seeks to maximize $u_i(z) = x'_i + \theta_i \ln (g')$, given an exogenous status quo $g$ and unanimity rule. Its one-shot problem analogous to (E2) is

$$\pi^i(g) \in \text{arg max}_{z=(g',x'_H,x'_L)\in \mathcal{B}} x'_i + \theta_i \ln (g')$$

s.t. $x'_j + \theta_j \ln (g') \geq K_j(g)$, where $K_j(g) = \ln (g)$.

PROPOSITION 2: In the one-period model with mandatory public spending and discretionary transfers, the unique equilibrium proposal strategy for party $i \in \{H, L\}$ is
\[ \gamma^i(g) = \begin{cases} \theta_i & \text{for } g \leq \theta_i, \\ \theta_H + \theta_L & \text{for } \theta_i \leq g \leq \theta_H + \theta_L, \\ \theta_H + \theta_L & \text{for } \theta_H + \theta_L \leq g \leq 1, \end{cases} \]

and \( \chi^i_j(g) = 1 - \gamma^i(g) - \chi^i_j(g) \).

We relegate the proof of Proposition 2 to the Appendix. Henceforth all omitted proofs are in the Appendix unless otherwise indicated. We illustrate \( \gamma^i(g) \) in Figure 2 for the one-period problem.\(^{17}\)

Notice that when the status quo level of the public good is below proposer \( i \)'s static ideal \( \theta_i \), proposer \( i \) has a constant choice of \( \gamma^i(g) \) equal to its static ideal. Intuitively, when the status quo is below some threshold, the responder’s acceptance constraint does not bind, and hence the proposer is able to set its ideal level of the public good and extract the remainder of the budget as a transfer for itself.\(^{18}\) When

\(^{17}\)Here we analyze a one-period problem for an exogenous status quo \((g, 0, 0)\). The equilibrium outcome in the infinite-horizon problem under an exogenous status quo \((g, 0, 0)\) is the repetition of this one-period solution since there is no dynamic link between choices today and future outcomes. This result extends to more general exogenous status quos.

\(^{18}\)If there is no lower bound on transfers, then the responder’s acceptance constraint is always binding (except when \( g = 0 \)) and the efficient level of the public good is chosen. We find it reasonable to have a lower bound on transfers given property rights. With any lower bound, there are equilibrium proposals that do not involve the efficient level of the public good even when the responder’s acceptance constraint binds.
the status quo is above this threshold, the responder’s acceptance constraint binds. For some intermediate range of the status quo, it is optimal for the proposer to maintain the level of the public good at the status quo and extract the remaining budget as a transfer. For status quos above the efficient level \( \theta_H + \theta_L \), since the sum of the marginal benefit of the public good is lower than the marginal benefit of transfers, the proposer does best by lowering the level of the public good to the efficient level, giving the responder a transfer to make the responder indifferent, and extracting the remainder of the budget for itself. Hence \( \gamma_i(g) \) is constant at the efficient level when the status quo is above the efficient level.

As we can see in the one-period model, the equilibrium level of public good spending under mandatory programs is higher than under discretionary programs and is strictly so when the status quo is sufficiently high. In fact the efficient level of the public good is achieved for status quos equal or above the efficient level. A higher level of the public good is achieved because the payoff the responder receives if it rejects the proposal is higher for any status quo than it is under discretionary programs. If the status quo payoff is high enough, the responder’s acceptance constraint binds, and the proposer chooses to set the level of the public good higher than its static ideal. This is the status quo effect.

Given that the equilibrium strategies and hence the payoffs in the one-period problem take different functional forms for different regions, the analysis of the \( T \)-period problem, even for \( T = 2 \), is cumbersome. Partly because of this, we do not analyze a \( T \)-period problem. Rather, we analyze the infinite-horizon problem by exploiting the recursive structure. We show that in the infinite-horizon model, the status quo effect leads to higher levels of public good spending than in the static model because of dynamic considerations.

**B. The Infinite-Horizon Model**

Now consider the infinite-horizon model. From the equilibrium conditions (E2), it must be the case that, for all \( i, j \in \{H, L\}, j \neq i \) and any status quo \( g \), the proposal \( \pi_i(g) \) is a solution to the following maximization problem,

\[
\pi_i(g) \in \arg \max_{z=(g', x_H^i, x_L^j) \in B} x_j^i + \theta_j \ln(g') + \delta \left[ p V_i(g') + (1 - p) W_i(g') \right]
\]

s.t.
\[
x_j^i + \theta_j \ln(g') + \delta \left[ (1 - p) V_j(g') + p W_j(g') \right] \geq K_j(g),
\]

where \( V_i \) and \( W_i \) satisfy (E3) and

\[
K_j(g) = \theta_j \ln(g) + \delta \left[ (1 - p) V_j(g) + p W_j(g) \right].
\]

We construct equilibria by the “guess and verify” method. The form of the parties’ equilibrium strategies in the one-period model are a natural starting place to conjecture the solution to the infinite-horizon model; however, we expect the solution to the infinite-horizon model to take into account continuation strategies and payoffs. We provide here some brief intuition about how this may affect the equilibrium. Consider the choice of the proposer when the responder’s constraint is not binding.
In the one-period model, the proposer chooses its static ideal. In the infinite-horizon model the proposer takes into account that it may not be the proposer in the next period; hence it may wish to provide insurance for itself by setting the level of the public good above its static ideal to raise its status quo payoff in case it becomes the responder.

This insurance effect appears to have the desirable property that it increases the equilibrium level of the public good compared to the underprovision that occurs with discretionary spending, but is it possible that it causes parties to increase the level of the public good above the efficient level? The answer is yes for some parameter values. In particular, define the level of polarization as the ratio $\theta_H/\theta_L$. Below, we divide the analysis of the infinite-horizon model into the low-polarization case and the high-polarization case. In the case of low polarization we show that the insurance effect leads each party to propose levels of public good spending that are higher than what it proposes when such spending is discretionary, but there is no overprovision in equilibrium. In the high-polarization case we observe overprovision.

**Low-Polarization Case.**—We show that in the low-polarization case, there exists an equilibrium that somewhat resembles the one-period solution. When the status quo is below some threshold, each proposer proposes a constant level of the public good to maximize its dynamic payoff—we call this the proposer’s dynamic ideal. Party $L$’s dynamic ideal is the same as its static ideal, but party $H$’s dynamic ideal is above its static ideal. When the status quo is above the threshold, the responder’s constraint binds and the proposer proposes a level of the public good above its dynamic ideal to make the responder willing to accept. Hence equilibrium levels of the public good are higher in the infinite-horizon model than in the one-period model, and higher than with discretionary public good spending. Accordingly, any steady state level of the public good is higher with mandatory programs than with discretionary programs.

**PROPOSITION 3:** With mandatory public spending and discretionary transfers, if polarization is sufficiently low, then there exists an equilibrium such that

(i) equilibrium levels of the public good are higher than with discretionary public good spending; specifically, for $i, j \in \{ H, L \}, j \neq i$

\[
\gamma^i(g) = \begin{cases} 
  g^* & \text{for } g \leq g^*_i, \\
  g & \text{for } g^*_i \leq g \leq \theta_H + \theta_L, \\
  \theta_H + \theta_L & \text{for } \theta_H + \theta_L \leq g,
\end{cases}
\]

\[
\chi^i_j(g) = \begin{cases} 
  0 & \text{for } g \leq \theta_H + \theta_L, \\
  \frac{\theta_j(1 - \delta p) - \theta_i \delta (1 - p)}{(1 - \delta)(1 + \delta - 2\delta p)} \ln \left( \frac{g}{\theta_H + \theta_L} \right) & \text{for } \theta_H + \theta_L \leq g,
\end{cases}
\]

and $\chi^i_i(g) = 1 - \gamma^i(g) - \chi^i_j(g)$, where
(7) \[ g^*_L = \theta_L \quad \text{and} \quad g^*_H = \frac{1 + \delta - 2\delta \rho}{1 - \delta \rho} \theta_H; \]

(ii) the set of steady-state public good levels is \([g^*_H, \theta_H + \theta_L]\).

Equation (7) characterizes each party’s dynamic ideal \(g^*_i\). A necessary condition for an equilibrium to exist with proposal strategies given in Proposition 3 is that \(g^*_H \leq \theta_H + \theta_L\). By equation (7), this is satisfied if \(\frac{\theta_H}{\theta_L} \leq \frac{(1 - \rho p)}{[\delta(1 - p)]}\). Since this condition implies that the parties’ preferences regarding the value of public good are sufficiently similar, we call this the “low-polarization” case. Note that \((1 - \rho p)/[\delta(1 - p)]\) is increasing in \(p\) and decreasing in \(\delta\) and therefore the low-polarization case can be alternatively interpreted as high persistence of power and impatient parties.

We provide an example of numerical output from value function iterations in Figures 3 and 4. Figure 3 illustrates the parties’ proposal strategies for the public good, and Figure 4 illustrates the parties’ proposal strategies for transfers. These are consistent with the characterization in Proposition 3.19

The parties’ dynamic ideals reflect the insurance effect discussed at the beginning of this subsection. In particular, it says that party \(L\)’s dynamic ideal \(g^*_L\) is equal to its static ideal \(\theta_L\), while party \(H\)’s dynamic ideal \(g^*_H = \theta_H(1 + \delta - 2\delta \rho)/(1 - \delta \rho)\) is strictly higher than its static ideal \(\theta_H\). To understand this result, note that the proposer’s choice of the public good level has a static effect on the current-period payoff and a dynamic effect on the continuation payoff because it determines next period’s status quo. Furthermore, the dynamic effect creates two competing incentives for the incumbent: the incentive to raise the public good level for fear that the opposition party comes into power next period, and the incentive to lower the public good level to lower the bargaining power of the opposition party if the incumbent stays in power next period. If polarization is low, the dynamic effect of party \(L\)’s proposal is zero because even if party \(H\) becomes the proposer next period, it would choose its

19 When polarization is sufficiently low, all numerical output we have obtained are consistent with the characterization in Proposition 3.
dynamic ideal, which is sufficiently high. On the other hand, party H is indeed concerned that party L would set the level of public good too low should party L come into power, and the insurance incentive arising from this dynamic concern leads party H to propose \( g_H^* \) strictly higher than its static ideal \( \theta_H \).

We provide a sketch of the derivation of \( g_L^* \) and \( g_H^* \) to help with this intuition. Given that \( g_L^* \) and \( g_H^* \) are the parties’ dynamic ideals, they maximize the parties’ dynamic payoffs unconstrained by the responder. Specifically, define \( f_i(g) \) as party \( i \)'s dynamic payoff when the public spending in the current period is \( g \) and party \( i \) receives the remaining surplus

\[
(8) \quad f_i(g) = 1 - g + \theta_i \ln(g) + \delta [pV_i(g) + (1 - p)W_i(g)].
\]

Party \( i \)'s dynamic ideal maximizes \( f_i(g) \).

First consider party H. Given that H values the public good more than L, we expect party H’s dynamic ideal to be higher than party L’s. Consider a status quo \( g \) between \( g_L^* \) and \( g_H^* \). Since party L would like to lower the level of the public good, this means that party H’s constraint as responder is binding, that is \( W_H(g) = K_H(g) \). With this observation and using the expression for \( K_H(g) \) in (6), we have that \( W_H(g) = [\theta_H \ln(g) + \delta(1 - p)V_H(g)]/(1 - \delta p) \). Substituting this expression for \( W_H(g) \) into (8) gives

\[
f_H(g) = 1 - g + \frac{1 + \delta - 2\delta p}{1 - \delta p} \theta_H \ln(g) + \frac{\delta(p + \delta - 2\delta p)}{1 - \delta p} V_H(g).
\]

Taking the derivative gives

\[
f'_H(g) = -1 + \frac{1 + \delta - 2\delta p}{g(1 - \delta p)} \theta_H + \frac{\delta(p + \delta - 2\delta p)}{1 - \delta p} V'_H(g).
\]

To find \( V'_H(g) \), note that by the equilibrium strategies when the status quo is below party H’s dynamic ideal, H proposes its dynamic ideal and keeps the remainder as a transfer, and L accepts. This implies that \( V_H(g) \) is constant below party H’s dynamic ideal, and hence \( V_H(g) = 0 \) at \( g_H^* \). Setting \( f'_H(g) = 0 \) and solving for \( g \) gives \( g_H^* = \theta_H (1 + \delta - 2\delta p)/(1 - \delta p) \).

Now consider party L. Taking the derivative of \( f_L(g) \) gives \( f'_L(g) = -1 + \theta_L \ln(g) + \delta [pV_L(g) + (1 - p)W_L(g)] \). As discussed in the previous paragraph, when the status quo is below \( g_H^* \), party H’s proposal is constant, implying that below \( g_H^* \), party L’s dynamic payoff as the responder, \( W_L(g) \), is constant, and therefore \( W'_L(g) = 0 \) at \( g_L^* \). Since party H’s dynamic ideal is above party L’s, party H is willing to accept party L’s dynamic ideal when the status quo is below it, and hence for such status quos, party L’s dynamic payoff as proposer is constant, and \( V'_L(g) = 0 \) at \( g_L^* \). Setting \( f'_L(g) = 0 \) and solving for \( g \) gives \( g_L^* = \theta_L \).

\(^{20}\)To be precise, the left derivative of \( V_H \) at \( g = g_H^* \) is equal to zero, but the right derivative is not. We show formally in the proof of Lemma 4 in the Appendix that \( g_H^* = \theta_H (1 + \delta - 2\delta p)/(1 - \delta p) \). The same caveat applies to the discussion of \( g_L^* \) in the next paragraph.
High-Polarization Case.—Now suppose $\theta_H/\theta_L > (1 - \delta p)/[\delta(1 - p)]$, which we call the high-polarization case. Figures 5 and 6 illustrate an example of numerical output from value function iteration when this condition holds.

Figures 5 and 6 show equilibrium strategies that look different from the low-polarization case at first glance; however, upon further examination, we find parallels. First consider the strategy illustrated for party $L$. This strategy is in fact similar to party $L$’s strategy in the low-polarization case: at low levels of the status quo, party $L$ chooses a constant level of the public good and gives party $H$ no transfer; for intermediate values of the status quo, party $L$ chooses the public good level equal to the status quo and again gives party $H$ no transfer; and for status quos above the efficient level ($\theta_H + \theta_L = 0.6$), the efficient level of the public good is chosen and now party $L$ gives party $H$ some transfer so that it accepts the proposal.

Recall that in Proposition 3, party $H$’s dynamic ideal is $g^*_H = \theta_H(1 + \delta - 2\delta p)/(1 - \delta p)$. The condition for high-polarization, $\theta_H/\theta_L > (1 - \delta p)/[\delta(1 - p)]$, necessitates that $g^*_H$ (which equals 0.67 for these parameter values) is now strictly above the efficient level, 0.6. It is not surprising that at low values of the status quo, below the point $g^*_H$ in Figure 5, party $H$ still chooses the public good spending to be equal to its dynamic ideal. Interestingly, Figure 5 shows that party $H$’s dynamic ideal is also chosen at very high levels of the status quo, which suggests that party $L$’s acceptance constraint is slack when the status quo is very high. The intuition for setting the level of the public good above the static ideal is the same as before: party $H$’s insurance motive dominates, but under high polarization, what is dynamically optimal for party $H$ is higher than even the efficient level.

Between $g^*_H$ and a higher threshold $\tilde{g}_H$, the level of public good proposed by party $H$ is between its dynamic ideal and the efficient level $\theta_H + \theta_L$. This is because the acceptance constraint for party $L$ binds and party $H$ cannot propose its dynamic ideal, but party $L$’s status quo payoff is low enough that party $H$ does not have to propose the efficient level. As the status quo increases, party $L$’s status quo payoff also increases, and party $H$ has to propose a level of the public good closer to the efficient level.
Between $\tilde{g}_H$ and $\theta_H + \theta_L$, the efficient level is proposed by party $H$. In this range, party $L$’s status quo payoff is high enough that party $H$ finds it optimal to propose the efficient level of the public good and give party $L$ some transfer so that it consents to raising the level of the public good. Finally, between the efficient level and party $H$’s dynamic ideal, it is optimal for party $H$ to maintain the status quo since it is closer to party $H$’s dynamic ideal, and it satisfies party $L$’s constraint.

The strategies illustrated in Figures 5 and 6 have several interesting implications. As in the low-polarization case, the equilibrium levels of public good spending are higher than with discretionary public good spending. Indeed, when the status quo is either sufficiently low or high, party $H$ proposes a level of public good spending above the efficient level (overprovision), but this is a transient state: the unique steady state is the efficient level of public good spending $\theta_H + \theta_L$. We summarize these properties in the next proposition.

**PROPOSITION 4:** In the high-polarization case with mandatory public spending and discretionary transfers, if polarization and parties’ values of the public good are not too high, then there exists an equilibrium in which

(i) equilibrium levels of the public good are higher than with discretionary public good spending;

(ii) there may be overprovision of the public good: specifically, if the status quo is sufficiently low or sufficiently high, then party $H$ proposes public good spending equal to its dynamic ideal $\tilde{g}_H^* = \theta_H (1 + \delta - 2\delta p) / (1 - \delta p)$, which is higher than the efficient level $\theta_H + \theta_L$;

(iii) the unique steady state level of the public good is the efficient level $\theta_H + \theta_L$.

Due to space limitations, the proof of Proposition 4 is in the online Appendix.

We next discuss equilibrium dynamics.

**Equilibrium Dynamics.**—In the low-polarization case, the strategies in Proposition 3 imply that, starting from a level of the public good below the efficient level, the steady state is still below the efficient level, but above what would be implemented with only discretionary programs. For example, if the initial status quo is below $g_L^* = \theta_L$ and party $L$ is the initial proposer, party $L$ chooses $\gamma_L(g) = \theta_L$ and this level persists until party $H$ next comes to power. When party $H$ is next in power, party $H$ sets a higher level of the public good $\gamma_H^*(g) = g_H^* \leq \theta_H + \theta_L$, and the public good spending remains at this level independent of who comes to power, that is, it is a steady state. For status quos between

---

21 We give a complete characterization in the high-polarization case when polarization and parties’ values of the public good are not too high. We discuss what happens for other parameters values in the high-polarization case and provide numerical examples in the online Appendix. All numerical output we have obtained in the high-polarization case are consistent with the characterization in Proposition 4.

22 Unlike the case of discretionary public spending, in the case of mandatory public spending for both high and low polarization, sustaining the efficient level of the public good with the threat of reverting to the Markov equilibrium we characterize is not an equilibrium. For example, if the status quo is at the dynamic ideal of party $L$, then party $L$ prefers maintaining the status quo to proposing the efficient level and therefore has an incentive to deviate.
$g^*_H$ and $\theta_H + \theta_L$, the level of public good spending is maintained at the status quo level, independent of which party is in power. Hence, any level of public good spending in this range is a steady state. Starting from a level of the public good above the efficient level, the steady state is at the efficient level. This is because when the status quo is above the efficient level, parties find it optimal to reduce spending on the public good to the efficient level, but once public good spending is at the efficient level, any allocation that exhausts the budget is on the Pareto frontier, that is, any proposal that improves the payoff of the proposer must reduce the payoff of the responder. Because public good spending is mandatory, the responder’s bargaining power prevents the proposer from reducing its payoff, and hence this is a steady state.

In the high-polarization case, Proposition 4 says that the only steady state involves public good spending equal to the efficient level $\theta_H + \theta_L$. The dynamics leading to this unique steady state may be nonmonotone. As seen in Figure 5, if the initial status quo is below $\hat{g}_H$ and party $L$ is the initial proposer, party $L$ chooses $\gamma_L(g) \in [\theta_L, \hat{g}_H]$ and this level persists until party $H$ next comes to power. When party $H$ is next in power, party $H$ sets a higher level of the public good $\gamma_H(g) \in [\theta_H + \theta_L, g^*_H]$, and the public good spending remains at this level until party $L$ next comes to power. When party $L$ returns to power, it finds it optimal to reduce the level of the public good to the efficient level and compensate party $H$ by providing transfers. It is the anticipation of these transfers that provided an incentive for party $H$ to propose a level of public spending above the efficient level when the state was low. Once the efficient level of public good spending is reached, it is sustained.

Proposition 3 says that in the equilibrium we constructed, the set of steady states is $[g^*_H, \theta_H + \theta_L]$ in the low-polarization case, and Proposition 4 says that it is the singleton $\{\theta_H + \theta_L\}$ in the high-polarization case. In the next proposition, we show that there are no other steady states in any other equilibrium under certain conditions.

Suppose $\sigma$ and $(V_H, W_H, V_L, W_L)$ is an equilibrium strategy-payoff pair. Let $G^*$ denote the set of steady states; that is, for any $g \in G^*$, $\gamma_i(g) = g$ for $i \in \{H, L\}$. Let $G$ denote the set of public good spending levels $g$ such that the acceptance constraint binds when the status quo is $g$, regardless of who the responder is.

**Proposition 5:** Let $g \in G^*$, and suppose that (i) $V_H$ and $V_L$ are differentiable on an open set $C$ such that $g \in C \subset G$, and (ii) the responders’ acceptance constraints satisfy Kuhn-Tucker Constraint Qualification. Then $g \in [g^*_H, \theta_H + \theta_L]$ in the low-polarization case, and $g = \theta_H + \theta_L$ in the high-polarization case.

**Comparative Statics.**—In the high-polarization case, the set of steady states is a singleton and only depends on $\theta_H$ and $\theta_L$. We next discuss comparative statics on the set of steady states in the low-polarization case. Since the highest steady state is constant at the efficient level, comparative statics on the set of steady states is driven by comparative statics on the lowest steady state, which is given by party $H$’s dynamic ideal level of the public good $g^*_H$.

**Proposition 6:** In the low-polarization case, the lowest steady state $g^*_H$ is decreasing in the persistence of power $p$ and increasing in the discount factor $\delta$. 

Proposition 6 holds because the derivative of $g_H^*$ with respect to $p$ is negative and with respect to $\delta$ is positive. The intuition for this result is simple. Dynamic considerations create incentives for party $H$ to set a level of the public good above its static ideal to increase its status quo payoff in the event that it loses (proposing) power. As party $H$ becomes more confident that it will still be in power in the next period, its incentive to insure itself decreases, and hence it sets a level of the public good closer to its static ideal, knowing that it will likely be able to set the same level in the next period without giving transfers to the other party. Similarly, as party $H$’s discount factor increases, it puts more weight on future payoffs, and hence is more sensitive to being out of power in the future. To insure itself against power fluctuations, it increases the level of the public good in the current period. Hence less persistence in political power or more patience results in steady states closer to the efficient level.

In several papers of dynamic political economy (see, for example, Alesina and Tabellini 1990; Besley and Coate 1998; and Azzimonti 2011), higher persistence of power reduces policy distortions because with a higher probability of reelection, current policymakers better internalize the future consequences of today’s policy choice. In contrast, we show that higher persistence of power reduces efficiency with mandatory programs. The reason for the difference is that here the state variable (the status quo) affects the future bargaining environment (as determined by the institution governing spending rules), whereas in previous work, the out-of-power party has no bargaining power and the state variable affects the future economic environment. This distinction underscores the significance of considering alternative institutional arrangements in understanding economic policy outcomes.

VI. Efficiency Implications of Mandatory Programs

One objective of this paper is to examine the efficiency implications of mandatory programs. We have already shown in Proposition 3 and Proposition 4 that mandatory programs raise the level of spending on the public good and improve the efficient provision of public good compared to discretionary programs. In this section we further explore how mandatory programs affect parties’ welfare. The next proposition shows that mandatory programs improve the ex ante welfare of party $H$. More surprisingly, under some conditions they also improve the ex ante welfare of party $L$.

PROPOSITION 7: Suppose it is equally likely ex ante for either party to become the proposer. Then party $H$’s expected steady-state payoff is higher when public good spending is mandatory than when it is discretionary. Moreover, in the low-polarization case, party $L$’s expected steady-state payoff is higher when public good spending is mandatory than when it is discretionary if parties are sufficiently patient and the persistence of power is sufficiently low.
Intuitively, mandatory programs provide payoff smoothing for the parties; that is, the difference between each party’s payoff when in power and when out of power is smaller under mandatory programs. When the persistence of power is low and parties are patient, this benefit is high, making the parties better off. This ex ante Pareto improvement may explain why many countries have enacted mandatory programs.

VII. All Spending Mandatory

So far we have analyzed the case in which spending on the public good is mandatory and transfers are discretionary. An important finding in this case is that mandatory programs improve the efficient provision of the public good. Does this result hold under other rules governing mandatory and discretionary programs? To address this question, in this section we consider the case in which all spending is mandatory; that is, \( \zeta(b) = b \) for all \( b \in \mathcal{B} \). In this case the payoff-relevant state is last period’s budget \( b = (g, x_H, x_L) \). We show that provision of the public good is still improved relative to the case without mandatory programs, demonstrating the robustness of our main result.

We establish that when all spending is mandatory, the set of steady states coincides with the set of static Pareto efficient allocations, and once a steady state is reached, the sequence of allocations that follows is dynamically Pareto efficient. We define a static Pareto efficient allocation as the analog of a dynamic Pareto efficient allocation defined in Section II. Formally, a static Pareto efficient allocation solves the following problem for some \( \bar{U} \in \mathbb{R} \):

\[
\max_{b \in \mathcal{B}} u_L(b) \\
\text{s.t. } u_H(b) \geq \bar{U}.
\]

**PROPOSITION 8:** In the infinite-horizon game with all mandatory spending, any steady state is efficient. Specifically, (i) a budget \( (g, x_H, x_L) \) is a steady state if and only if it is a static Pareto efficient allocation; and (ii) the infinite repetition of a steady state is a dynamic Pareto efficient allocation.

The proof of Proposition 8 is in the online Appendix. To gain some intuition for part (i), note that when all spending is mandatory, given a status quo \( s \), responder \( j \)'s dynamic payoff \( W_j(s) \geq u_j(s)/(1 - \delta) \) since it can maintain the status quo \( s \) by rejecting any proposal not equal to \( s \), and proposing to maintain \( s \) when it comes to power in the future. We show that an infinite repetition of a static Pareto efficient allocation is dynamically Pareto efficient. This implies that when the status quo \( s \) is statically Pareto efficient, proposer \( i \)'s dynamic payoff is no greater than \( u_i(s)/(1 - \delta) \) since the responder \( j \) can guarantee a payoff of \( u_j(s)/(1 - \delta) \). Note also that for any status quo \( s \), proposer \( i \)'s dynamic payoff \( V_i(s) \geq u_i(s)/(1 - \delta) \) since it can maintain the status quo by proposing \( s \) when in power, and rejecting any proposal other than \( s \) if party \( j \) comes to power in the future. Hence, when \( s \) is statically Pareto efficient, proposing to maintain the status quo is a best response for the proposer, and therefore \( s \) is a steady state. Conversely, if \( s \) is not a static Pareto efficient allocation, then proposing to maintain \( s \) is not a best response for the proposer
because it can achieve a higher dynamic payoff while satisfying the responder’s acceptance constraint. Hence \( s \) is not a steady state in this case.\(^{24}\) Part (ii) follows immediately from part (i) since an infinite repetition of a static Pareto efficient allocation is dynamically Pareto efficient. Proposition 8 contrasts with the case of discretionary public spending in which the equilibrium allocation is not dynamically Pareto efficient.

We provide illustrations of proposal strategies from value function iteration in Figure 7. In these illustrations we fix the status quo values of \( x_H \) and \( x_L \) to be zero and examine how the proposals vary with the value of the status quo level of the public good \( g \).\(^{25}\)

We make the following observations from these illustrations. First, when \( g \) is very close to zero, party \( i \) proposes its ideal allocation, \( g' = \theta_i, x_i' = 1 - \theta_i, x_j' = 0 \). Second, the equilibrium level of the public good is increasing in the status quo level of the public good because it raises the responder’s bargaining power. Above some threshold level of \( g \), the equilibrium level of the public good is at the efficient level. Interestingly, these thresholds are below the efficient level, so for some status quos below the efficient level, the proposer raises the spending on the public good to the efficient level. The reason is that for the proposer to extract the rest as transfer for itself (which becomes the status quo in the next period since all spending is mandatory), it must raise the level of the public good so that the responding party accepts. This implies that the level of the public good proposed in equilibrium is higher than the status quo for all levels of the status quo below the efficient level. In particular, if the status quo level of the public good is party \( i \)’s ideal \( \theta_i \), party \( i \) selects a level of the public good higher than its ideal. All allocations illustrated are Pareto efficient, and hence are steady states.

\(^{24}\) It is straightforward to show that the following are the static Pareto efficient allocations: \((g, x_H, x_L) \in \mathcal{B}\) where \( g = \theta_H + \theta_L, g + x_H + x_L = 1, \) or \( g \in [\theta_H, \theta_H + \theta_L], x_H = 1 - g, \) or \( g \in [\theta_L, \theta_H + \theta_L], x_L = 1 - g \).

\(^{25}\) We provide illustrations for positive values of \( x_H \) and \( x_L \) in the online Appendix for comparison, and the equilibrium strategies are qualitatively similar.
Note that in the illustration, the equilibrium level of public good spending is higher than in the discretionary case. As discussed in the previous paragraph, the reason that it is higher is because the status quo payoff of the out-of-power party is higher with all programs mandatory than with all programs discretionary. When mandatory programs give the responding party a sufficiently high status quo payoff, the status quo effect leads to increased spending on the public good as long as the joint marginal benefits from the public good is greater than the marginal benefit from transfers. Hence it is the status quo effect that leads to higher equilibrium levels of the public good.

VIII. Concluding Remarks

In this paper we analyze a model of dynamic bargaining between two political parties over the allocation of a public good and private transfers to understand the efficiency implications of mandatory programs. We find that mandatory programs mitigate the problem of underprovision of the public good compared to discretionary programs because it raises the bargaining power of the out-of-power party. In the case of mandatory public spending, the mandatory program provides a channel for parties to insure themselves against power fluctuations. As a result, it provides payoff smoothing for the parties, and this leads to higher ex ante dynamic payoffs for both parties, even the one that places a low value on the public good, when the parties are sufficiently patient, not too polarized, and persistence of power is sufficiently low.

Several extensions seem promising for future research. First, in this paper, we provide some insight into how different status quo rules affect budget negotiations, but since there are other rules governing how the status quo evolves beyond the ones explored in this paper, an interesting question is what is the optimal status quo rule. Separately, if the choice of mandatory versus discretionary programs is endogenous, what would be the outcome? One way of endogenizing mandatory spending is by allowing parties to choose the number of periods for which the program is mandatory. This can be thought of as modeling sunset provisions as considered in a finite horizon setting in Auerbach (2006).

The persistence of power is parameterized by $p$, the probability that the proposer last period continues to be the proposer this period, and for simplicity, we assume it to be exogenous in our model. Since success in bringing home “pork” typically results in more favorable electoral outcomes, a second interesting extension is to consider how the efficiency implications of mandatory programs change if power persistence is endogenously determined by the policy choice as in Azzimonti (2011) and Bai and Lagunoff (2011).

In our model, the size of the budget to be allocated in each period is fixed. Another extension is to investigate the effect of mandatory programs if the size of the budget is endogenous and determined by policy choice. One example is to consider a model in which the portion of the budget not consumed in the current period is added to the next period’s budget. Alternatively, one might consider the effect of mandatory programs in a neoclassical growth model à la Battaglini and Coate (2008).

\footnote{In this alternate model, when parties disagree, there is no waste as resources not allocated this period become part of the next period’s budget. This is different from the assumption in the current paper.}
Our model presents a stylized political setting with two parties deciding on a budget. Parties may also be thought of as a collection of legislators who potentially have independent interests. In that case, a model of multiple legislators may be considered with majority rules other than unanimity as in Baron and Ferejohn (1989). Exploring the efficiency implications of mandatory programs in this setting is another possible direction for future research.

Finally, although parties place different values on the public good, each party’s value stays constant over time in our model. If the values of the public good fluctuate over time stochastically, then we expect mandatory programs to have other interesting effects absent in the model with deterministic values. For example, a high level of public good spending that is efficient in times when the public good is especially valuable becomes inefficient when the value of the public good decreases, and the inertia created by the mandatory program may lead to overprovision of the public good. In some preliminary analysis of a model in which the public good has the same value to both parties but fluctuates stochastically over time, we find that overprovision of the public good can happen when the value of the public good is low but the status quo is high. We plan to pursue this extension and others mentioned above in future work.

APPENDIX

A. Proof of Proposition 2

Party $i$’s Lagrangian for this problem is

$$L_i = x_i' + \theta_i \ln(g') + \lambda_1 [1 - g' - x_i' - x_j'] + \lambda_2 [x_j' + \theta_j \ln(g') - K_j(g)],$$

where $K_j(g) = \theta_j \ln(g)$. The first order conditions are $g'$, $x_i'$, $x_j'$, $\lambda_1$, $\lambda_2 \geq 0$ and

(A1) \[ \frac{\theta_i}{g'} - \lambda_1 + \frac{\theta_j}{g'} \leq 0, \quad \frac{\theta_i}{g'} - \lambda_1 + \frac{\theta_j}{g'} g' = 0, \]

(A2) \[ 1 - \lambda_1 \leq 0, \quad [1 - \lambda_1]x_i' = 0, \]

(A3) \[ -\lambda_1 + \lambda_2 \leq 0, \quad [-\lambda_1 + \lambda_2]x_j' = 0, \]

(A4) \[ 1 - g' - x_i' - x_j' \geq 0, \quad [1 - g' - x_i' - x_j'] \lambda_1 = 0, \]

(A5) \[ x_j' + \theta_j \ln(g') - K_j(g) \geq 0, \quad [x_j' + \theta_j \ln(g') - K_j(g)] \lambda_2 = 0. \]

First note that $\lambda_1 \geq 1$ by (A2). Hence, (A4) implies that $1 - g' - x_i' - x_j' = 0$. Next note that $0 < g' < 1$. If $g' = 0$, then (A1) is violated. If $g' = 1$, then $\lambda_2 = \frac{\lambda_1 - \theta_i}{\theta_j}$ by (A1). Combining this value of $\lambda_2$ with (A3) gives $\lambda_1 = \frac{\lambda_1 - \theta_i}{\theta_j} \leq \lambda_1$. Rearranging implies $\lambda_1 \leq \frac{\theta_i}{1 - \theta_j}$. For this to be consistent with (A2) we need $\theta_i + \theta_j \geq 1$, a contradiction.

Since $g' < 1$ implies $x_i' = x_j' = 0$ is not optimal, there are now four cases to consider.
• $\lambda_2 = 0$: Since $\lambda_1 > 0$, (A3) implies that $x'_j = 0$. Combining this with $g' < 1$, we have $x'_i > 0$. By (A2), $x'_i > 0$ implies that $\lambda_1 = 1$. Combined with (A1), this implies that $g' = \theta_i$, $x'_i = 1 - \theta_i$, and $x'_j = 0$. For the inequality in (A5) to hold, we need $g \leq \theta_j$.

• $\lambda_2 > 0$, $x'_i > 0$ and $x'_j > 0$: Then $\lambda_1 = \lambda_2 = 1$. Together with (A1), (A4), and (A5), this implies that

\[
\begin{align*}
g' &= \theta_H + \theta_L, \\
x'_i &= 1 - \theta_L - \theta_H - K_j(g) + \theta_j \ln(\theta_H + \theta_L), \\
x'_j &= K_j(g) - \theta_j \ln(\theta_H + \theta_L).
\end{align*}
\]

Since $0 \leq x'_i \leq 1$ and $0 \leq x'_j \leq 1$, for this to be a valid solution we need $0 \leq K_j(g) - \theta_j \ln(\theta_H + \theta_L) \leq 1 - \theta_H - \theta_L$, which holds if $g \geq \theta_H + \theta_L$.

• $\lambda_2 > 0$, $x'_i > 0$ and $x'_j = 0$: Then (A5) implies that $g' = g$. Since $x'_i > 0$, $\lambda_1 = 1$, and (A1) gives $g' = \theta_i + \lambda_2 \theta_j$. Since $0 < \lambda_2 < \lambda_1 = 1$, it follows that this is a valid solution only when $\theta_i < g \leq \theta_H + \theta_L$.

• $\lambda_2 > 0$, $x'_i = 0$ and $x'_j > 0$: Then (A3) gives $\lambda_1 = \lambda_2$ and (A1) gives $g' = \frac{\theta_i}{\lambda_1} + \theta_j > \theta_j$. Since $\lambda_2 > 0$, by (A5), $1 - g' + \theta_j \ln(g') = \theta_j \ln(g)$, which is impossible since $g' > \theta_j$.

To summarize, we have the solution given in Proposition 2.

\section*{B. Proof of Proposition 3}

To prove the first part of Proposition 3 we characterize an equilibrium strategy-payoff pair for which proposal strategies are as given in Proposition 3. We do this by the “guess and verify” method. We conjecture an equilibrium strategy-payoff pair $\sigma = ((\pi^H, \alpha^H), (\pi^L, \alpha^L))$ and $(V_H, W_H, V_L, W_L)$ with the following properties:

\begin{enumerate}
  \item[(G1)] There exist $g^*_L$ and $g^*_H$ with $g^*_L < g^*_H < \theta_H + \theta_L$ such that $g^*_i \in \arg\max f_i(g)$ for $i \in \{H, L\}$ and if $g \leq g^*_i$, then $\pi^i(g) = \pi^i(g^*_i)$, and specifically $\gamma^i(g) = g^*_i$.
  \item[(G2)] If $g \in [g^*_i, \theta_H + \theta_L]$, then $\gamma^i(g) = g$ and $W_j(g) = K_j(g)$ for $i, j \in \{H, L\}$ with $i \neq j$.
  \item[(G3)] For any $i, j \in \{H, L\}$ with $j \neq i$, if $g \geq \theta_H + \theta_L$, then $\gamma^i(g) = \theta_H + \theta_L$, $W_j(g) = K_j(g)$, and the proposer’s equilibrium payoff $V_i(g)$ takes the form $V_i(g) = C_j \ln(g) + D_j$.
\end{enumerate}

Suppose $\sigma = ((\pi^H, \alpha^H), (\pi^L, \alpha^L))$ and $(V_H, W_H, V_L, W_L)$ is an equilibrium strategy-payoff pair that satisfies (G1)-(G3). In the next few lemmas we establish some properties of $\sigma$ and $(V_H, W_H, V_L, W_L)$.

First consider the proposer’s problem (4) without imposing the responder’s acceptance constraint (5). Recall for proposer $i$ this implies maximizing
LEMMA 3: \[ f_i(g') = 1 - g' + \theta_i \ln(g') + \delta [p V_i(g') + (1 - p) W_i(g')] \]

Since the status quo \( g \) enters the problem only through the constraint (5), the proposer’s value function is independent of \( g \), and we denote proposer \( i \)'s highest payoff without the constraint (5) by \( V^*_i = \max_{g'} f_i(g') \). Also denote \( W_L(g^*_H) \) by \( W^*_L \) and denote \( W_H(g^*_L) \) by \( W_H^* \).

LEMMA 1: Under (G1), for all \( i, j \in \{H, L\} \) with \( j \neq i \), (i) if \( g \leq g^*_i \), then \( V_i(g) = V^*_i, \chi^*_i(g) = 1 - g^*_i, \chi'_i(g) = 0 \), and (ii) if \( g \leq g^*_j \), then \( W_i(g) = W^*_i \).

PROOF:
Part (i): By (G1), \( g^*_i \in \arg \max f_i(g) \). Since responder \( j \) accepts the proposal in which \( g' = g^*_i \), \( x'_i = 1 - g^*_i \), and \( x'_j = 0 \), it follows that \( V_i(g^*_i) \geq V^*_i \). Since \( V^*_i \) is proposer \( i \)'s highest payoff without the constraint (5), it follows that \( V^*_i \geq V_i(g) \) for any \( g \). Hence, \( V_i(g^*_i) = V^*_i, \chi^*_i(g^*_i) = 1 - g^*_i \), and \( \chi'_i(g^*_i) = 0 \). The rest of (i) follows immediately from (G1).
Part (ii) follows from the definition of \( W_i \) in (E3).

Now consider the case when the responder’s constraint is binding. Lemma 2 conveniently transforms the dynamic payoff for party \( i \) into one with a single value function \( V_i(g) \), rather than two—\( V_i(g) \) and \( W_i(g) \)—when party \( i \)'s constraint is binding.

LEMMA 2: If \( W_i(g) = K_i(g) \), then

\[
(A6) \quad W_i(g) = K_i(g) = \frac{1}{1 - \delta p} [\theta_i \ln(g) + \delta (1 - p) V_i(g)].
\]

PROOF:
Suppose \( W_i(g) = K_i(g) \). Then \( W_i(g) = \theta_i \ln(g) + \delta [(1 - p) V_i(g) + p W_i(g)] \). Rearranging gives (A6).

We characterize the dynamic payoffs over the range \( g \in [g^*_i, \theta_H + \theta_L] \).

LEMMA 3: Under (G1) and (G2), if \( g \in [g^*_L, g^*_H] \), then

\[
(A7) \quad V_L(g) = \frac{1}{1 - \delta p} [1 - g + \theta_L \ln(g) + \delta (1 - p) W_L^*],
\]

and if \( g \in [g^*_H, \theta_H + \theta_L] \), then

\[
(A8) \quad V_i(g) = \frac{(1 - \delta p)(1 - g)}{(1 - \delta)(1 + \delta - 2\delta p)} + \frac{\theta_i}{1 - \delta} \ln(g)
\]

for all \( i \in \{H, L\} \).

PROOF:
Under (G2), if \( g \in [g^*_i, \theta_H + \theta_L] \), then \( \gamma_i(g) = g \). Since the responder \( j \) accepts the proposal with \( g' = g \) and \( x'_i = 1 - g \) if the status quo is \( g \), this implies that \( \chi'_i(g) = 0 \) for \( g \in [g^*_i, \theta_H + \theta_L] \) and therefore
(A9) \[ V_i(g) = 1 - g + \theta_i \ln(g) + \delta[pV_i(g) + (1 - p)W_i(g)]. \]

By Lemma 1, if \( g \in [g \_L^*, g \_H^*] \), then \( W_L(g) = W_L^* \). Substituting in (A9) and rearranging terms, we get (A7). Under (G2), if \( g \in [g \_H^*, \theta_H + \theta_L] \), then \( W_i(g) = K_i(g) \) and by Lemma 2, equation (A6) holds. Substituting (A6) in (A9) and rearranging terms, we get (A8).

We now characterize \( g \_L^* \).

**LEMMA 4:** Under (G1) and (G2), \( g \_L^* = \theta_L \) and \( g \_H^* = \frac{1 + \delta - 2\delta p}{1 - \delta p} \theta_H. \)

**PROOF:**

We first show that \( g \_L^* = \theta_L \). Since \( V_i(g) \) and \( W_L(g) \) are constant for \( g \leq g \_L^* \) by Lemma 1, it follows that for \( g < g \_L^* \), \( \frac{\delta f_i(g)}{\delta g} = -1 + \frac{\theta_L}{g} \). If \( g \_L^* > \theta_L \), then \( f_i(\theta_L) > f_i(g \_L^*) \), contradicting that \( g \_L^* \in \text{arg max} \ f_i(g) \). Hence \( g \_L^* \leq \theta_L \). For \( g \in [g \_L^*, g \_H^*] \), \( V_i(g) = -\frac{1}{1 - \delta p} + \frac{\theta_L}{(1 - \delta p)g} \) by Lemma 3, and \( W'_i(g) = 0 \) by Lemma 1. Substituting these in \( f'_i(g) \), we get

\[
 f'_i(g) = -1 + \frac{\theta_L}{g} + \delta p V'_i(g) = \frac{1}{1 - \delta p} \left( -1 + \frac{\theta_L}{g} \right). 
\]

If \( g \_L^* < \theta_L \), then \( f_i(g \_L^*) < f_i(\theta_L) \) for any \( g \in (g \_L^*, \min\{\theta_L, g \_H^*\}) \), contradicting that \( g \_L^* \in \text{arg max} \ f_i(g) \). Hence, \( g \_L^* \geq \theta_L \). Since \( g \_L^* \leq \theta_L \) and \( g \_L^* \geq \theta_L \), it follows that \( g \_L^* = \theta_L \).

We next show that \( g \_H^* = \frac{1 + \delta - 2\delta p}{1 - \delta p} \theta_H \). If \( g \in (g \_L^*, g \_H^*) \), then \( V'_i(g) = 0 \) by Lemma 1 and \( W'_i(g) = \frac{\theta_H}{(1 - \delta p)g} \) by Lemma 2, and therefore

\[
(A10) \quad f'_i(g) = -1 + \frac{\theta_H}{g} + \delta(1 - p)W'_i(g) = -1 + \frac{(1 + \delta - 2\delta p)\theta_H}{(1 - \delta p)g}. 
\]

If \( g \_H^* > \theta_H \), then (A10) implies that \( f'_i(g) < 0 \) for \( g \in (\theta_H^*, g \_H^*) \), contradicting that \( g \_H^* \in \text{arg max} f_i(g) \). Hence \( g \_H^* \leq \theta_H^* \). If \( g \in (g \_H^*, \theta_H + \theta_L) \), then as shown in (A9), \( f_i(g) = V_i(g) \), and by (A8)

\[
(A11) \quad f'_i(g) = -\frac{1 - \delta p}{(1 - \delta)(1 + \delta - 2\delta p)} + \frac{\theta_H}{(1 - \delta)g}. 
\]

If \( g \_H^* < \theta_H \), then (A11) implies that \( f'_i(g) > 0 \) for \( g \in (g \_H^*, \theta_H^*) \), contradicting that \( g \_H^* \in \text{arg max} f_i(g) \). Hence \( g \_H^* \geq \theta_H^* \). Since \( g \_H^* \leq \theta_H^* \) and \( g \_H^* \geq \theta_H^* \), it follows that \( g \_H^* = \theta_H^* \).

Lemma 5 gives the proposer’s dynamic payoff over the remainder of the range of \( g \).
LEMMA 5: Under (G3),

\[
C_i = \frac{-(1 - \delta p)\theta_j + \delta(1 - p)\theta_i}{(1 - \delta)(1 + \delta - 2\delta p)},
\]

\[
D_i = \frac{(1 - \delta p)(1 - \theta_L - \theta_H + (\theta_H + \theta_L)\ln(\theta_H + \theta_L))}{(1 - \delta)(1 + \delta - 2\delta p)},
\]

for \( i, j \in \{H, L\} \) with \( j \neq i \).

PROOF:

Under (G3), for \( i \in \{H, L\} \), \( W_i(g) = K_i(g) \) for \( g \geq \theta_H + \theta_L \). By Lemma 2, \( W_i(g) = K_i(g) = \frac{\theta_i}{1 - \delta p} \ln(g) + \frac{\delta(1 - p)}{1 - \delta p} V_i(g) \). Consider any \( g \geq \theta_H + \theta_L \). Under (G3),

\[
\gamma_i'(g) = \theta_H + \theta_L \text{ and therefore }
\]

\[
V_i(g) = \chi_i'(g) + \theta_i \ln(\theta_H + \theta_L) + \delta[pV_i(\theta_H + \theta_L) + (1 - p)W_i(\theta_H + \theta_L)].
\]

After substituting for \( W_i(\theta_H + \theta_L) \), we have

\[
V_i(g) = \chi_i'(g) + \frac{1 + \delta - 2\delta p}{1 - \delta p} \theta_i \ln(\theta_H + \theta_L) + \frac{\delta(p + \delta - 2\delta p)}{1 - \delta p} V_i(\theta_H + \theta_L).
\]

Since the responder's acceptance constraint is binding at \( g \), we get

\[
\chi_i'(g) = K_i(g) - \frac{\theta_j}{1 - \delta p} \ln(\theta_H + \theta_L) - \frac{\delta(1 - p)}{1 - \delta p} V_j(\theta_H + \theta_L).
\]

Substituting \( K_j(g) = \frac{\theta_j}{1 - \delta p} \ln(g) + \frac{\delta(1 - p)}{1 - \delta p} V_j(g) \), \( \chi_i'(g) = 1 - \chi_j'(g) - \theta_L - \theta_H \), \( V_i(g) = C_i \ln(g) + D_i \), \( V_j(g) = C_j \ln(g) + D_j \) and matching the coefficients, we get (A12) and (A13).

By (G3), \( \gamma_i'(g) = \theta_H + \theta_L \) for all \( g \geq \theta_H + \theta_L \). To ensure that this holds in equilibrium, we need the responder to accept some feasible proposal with public spending equal to the efficient level for all \( g \geq \theta_H + \theta_L \). This is satisfied if and only if the responder would agree to bring the public spending down to \( \theta_H + \theta_L \) after receiving the rest of the surplus as private transfers. We derive a condition under which this holds in equilibrium. (We discuss what happens if the condition is violated in the online Appendix.)

LEMMA 6: Under (G1)-(G3), if

\[
(*) \quad 1 - (\theta_H + \theta_L) \geq \frac{\theta_H(1 - \delta p) - \theta_L(1 - p)}{(1 - \delta)(1 + \delta - 2\delta p)} (-\ln(\theta_H + \theta_L)),
\]

then \( \alpha_j'(g, (\theta_H + \theta_L, x_H, x_L)) = 1 \) when \( x_j = 1 - \theta_L - \theta_H, x_i = 0 \) for all \( g \geq \theta_H + \theta_L \).
PROOF:

Note that $\alpha^j(g, (\theta_H + \theta_L, x_H, x_L)) = 1$ with $x_j = 1 - \theta_L - \theta_H, x_i = 0$ is satisfied if

$$1 - (\theta_H + \theta_L) + \theta_j \ln(\theta_H + \theta_L) + \delta [(1 - p)V_j(\theta_H + \theta_L) + pW_j(\theta_H + \theta_L)] \geq K_j(g).$$

Substituting for $K_j(g)$ and $W_j(g)$ using Lemma 2 and substituting for $V_j(g) = C_j \ln(g) + D_j$ from Lemma 5 for $g \geq \theta_H + \theta_L$, the inequality simplifies to

$$A14 \quad 1 - (\theta_H + \theta_L) \geq \frac{\theta_j(1 - \delta p) - \theta_i \delta(1 - p)}{(1 - \delta)(1 + \delta - 2\delta p)} \left[ \ln(g) - \ln(\theta_H + \theta_L) \right].$$

Since the right-hand side of inequality (A14) is higher when $j = H$ than when $j = L$, it follows that if the inequality holds for $j = H$, then it holds for $j = L$ as well. Moreover, the right-hand side of (A14) is increasing in $g$, implying that if the inequality holds for $g = 1$, then it holds for all $g \geq \theta_H + \theta_L$. Substituting for $g = 1$ gives $(*)$.

We now establish the following result in the low-polarization case.

**LEMMA 7:** Suppose $\frac{\theta_H}{\theta_L} \leq \frac{1 - \delta p}{\delta(1 - p)}$ and condition $(*)$ holds. Then, there exists an equilibrium strategy-payoff pair $\sigma = ((\pi^H, \alpha^H), (\pi^L, \alpha^L))$ and $(V_H, W_H, V_L, W_L)$ that satisfies (G1)–(G3) and the proposal strategies are as given in part (i) of Proposition 3.

Due to space limitations, the proof of Lemma 7 is in the online Appendix. It involves conjecturing a pair of equilibrium strategies and payoffs, and showing that they satisfy (G1)–(G3) (and part (i) of Proposition 3) and equilibrium conditions (E1)–(E3).

Both the condition $\frac{\theta_H}{\theta_L} \leq \frac{1 - \delta p}{\delta(1 - p)}$ and condition $(*)$ hold when polarization is sufficiently low. In particular, it is easy to verify that condition $(*)$ holds strictly when $\theta_H = \theta_L$. By continuity, condition $(*)$ holds if polarization is sufficiently low. Then part (i) of Proposition 3 follows from Lemma 7.

The proof of Proposition 3 (ii) follows from inspecting strategies given in part (i).

C. Proof of Proposition 5

Fix $g \in \mathcal{G}$. First we show that $g \in \mathcal{G}$; that is, the responder’s acceptance constraint binds when the status quo is in $\mathcal{G}$. This follows immediately from the following claim:

**CLAIM 1:** For any $g \in \mathcal{G}^i$ and $i, j \in \{H, L\}$ with $i \neq j$, $\chi_j^i(g) = 0$.

**PROOF:**

Fix $g \in \mathcal{G}^i$. By definition of $\mathcal{G}^i$, $\gamma^i(g) = g$. Suppose to the contrary that $\chi_j^i(g) > 0$ for $j \neq i$. Let $\tilde{\pi}^i = (\tilde{\gamma}^i, \tilde{\chi}_H, \tilde{\chi}_L)$ be an alternative proposal strategy for player $i$ such
that \( \tilde{\pi}'(g') = \pi'(g') \) for \( g' \neq g \), \( \tilde{\gamma}'(g) = \gamma'(g), \tilde{\chi}'(g) = 0 \) and \( \tilde{\chi}'(g) > \chi'(g) \). Note that \( \tilde{\pi}' \) satisfies the responder’s acceptance constraint (5) when \( i \) is the proposer. Then \( \tilde{\pi}' \) yields the same payoff to player \( i \) for any \( g' \neq g \), and strictly higher payoff when the status quo is \( g \), contradicting that \( \pi' \) is an equilibrium proposal strategy.

Since \( g \in G \), we can simplify the proposer \( i \)'s maximization problem by using Lemma 2 to substitute for \( W_i \) and \( W_j \). Define the function \( h_i : B \to \mathbb{R} \) as

\[
h_i(g, x_H, x_L) = x_i + \frac{\theta_i}{1-\delta p} \ln(g) + \frac{\delta(1-p)}{1-\delta p} V_i(g).
\]

CLAIM 2: For any \( g \in G' \) and \( i \in \{H, L\} \),

\[
(A15) \quad V_i(g) = \max_{z = (g', x_H, x_L) \in B} x_i + \frac{1 - 2\delta p + \delta}{1-\delta p} \theta_i \ln(g') + \frac{\delta(p + \delta - 2\delta p)}{1-\delta p} V_i(g')
\]

s.t. \( h_i(z) \geq K_j(g), \ g' \in G \)

where \( K_j(g) = \frac{\theta_i}{1-\delta p} \ln(g) + \frac{\delta(1-p)}{1-\delta p} V_i(g) \).

PROOF:

By definition of \( G' \), the proposal \((g, \chi_H(g), \chi_L(g))\) is a solution to the maximization problem given in (4) and (5). By Claim 1, \( G' \subseteq G \), and so the proposal \((g, \chi_H(g), \chi_L(g))\) is also a solution to (4) and (5) when the maximization is over \( z = (g', x_H, x_L) \in B \) with \( g' \in G \). Since the acceptance constraint binds for any \( g \in G \), we use Lemma 2 to substitute for \( W_i \) and \( W_j \), resulting in the maximization problem given in Claim 2.

We are now ready to prove Proposition 5. Suppose \( h_H \) and \( h_L \) satisfy Kuhn-Tucker Constraint Qualification. The Lagrangian for party \( i \)'s problem, for \( i \in \{H, L\} \), is

\[
L_i = x_i' + \frac{1 - 2\delta p + \delta}{1-\delta p} \theta_i \ln(g') + \frac{\delta(p + \delta - 2\delta p)}{1-\delta p} V_i(g')
\]

\[
+ \lambda_{1i}[1 - x_i' - x_j' - g'] + \lambda_{2i}[x_j' + \frac{\theta_j}{1-\delta p} \ln(g') + \frac{\delta(1-p)}{1-\delta p} V_j(g') - K_j(g)]
\]

where \( j \in \{H, L\}, j \neq i \).

By the Kuhn-Tucker Theorem (see Takayama 1985, theorem 1.D.3), the first order necessary conditions for \((g', x_H', x_L')\) to be a solution to (A15) are \( \lambda_{1i} \geq 0, \lambda_{2i} \geq 0, g' \geq 0, x'_H \geq 0, x'_L \geq 0, \) and

\[
(A16) \quad 1 - \lambda_{1i} \leq 0, \quad [1 - \lambda_{1i}]x_i' = 0,
\]

\[
(A17) \quad -\lambda_{1i} + \lambda_{2i} \leq 0, \quad [-\lambda_{1i} + \lambda_{2i}]x_j' = 0,
\]
\( A20 \) \[ \theta_i (1 - 2\delta p + \delta) \frac{g' (1 - \delta p)}{g' (1 - \delta p)} + \frac{\delta (p + \delta - 2\delta p)}{1 - \delta p} \partial V_i \]

\[- \lambda_{1i} + \lambda_{2i} \left[ \frac{\theta_j}{g' (1 - \delta p)} + \frac{\delta (1 - p)}{1 - \delta p} \partial V_j \right] \leq 0, \]

\( A21 \) \[ \frac{\theta_j}{g' (1 - \delta p)} + \frac{\delta (p + \delta - 2\delta p)}{1 - \delta p} \partial V_i \]

\[- \lambda_{1i} + \lambda_{2i} \left[ \frac{\theta_j}{g' (1 - \delta p)} + \frac{\delta (1 - p)}{1 - \delta p} \partial V_j \right] g' = 0, \]

\( A22 \) \[ 1 - x_i' - x_j' - g' \geq 0 \quad [1 - x_i' - x_j' - g'] \lambda_{1i} = 0, \]

\( A23 \) \[ \frac{\partial V_i}{\partial g} = -\lambda_{2i} \frac{\partial K_j}{\partial g} = -\lambda_{2i} \left[ \frac{\theta_j}{g (1 - \delta p)} + \frac{\delta (1 - p)}{1 - \delta p} \partial V_j \right]. \]

By Claim 1, \( x_j' = 0 \). By (A16) \( \lambda_{1i} > 0 \), and so the feasibility constraint (A20) holds with equality. By the envelope theorem (see Takayama 1985, theorem 1.F.1), for \( i \in \{ H, L \} \), we have

\( A24 \) \[ \frac{\partial V_i}{\partial g} = \frac{\lambda_{2i} [\lambda_{2j} \theta_j \delta (1 - p) - \theta_j (1 - \delta p)]}{g [(1 - \delta p)^2 - \lambda_{2j} \lambda_{2j} \delta^2 (1 - p)^2]}, \]

for \( i, j \in \{ H, L \} \) with \( j \neq i \).

Since \( V_H \) and \( V_L \) are differentiable in an open set containing \( g \), it must be the case that \( g \in (0, 1) \). Since \( g \in G' \), this in turn implies that \( g' = g \in (0, 1) \). From \( g' > 0 \), it follows that (A18) must hold with equality for \( i, j \in \{ H, L \} \) and \( j \neq i \). From \( g' < 1 \), it follows that \( x_i' > 0 \), and hence \( \lambda_{1i} = 1 \) for \( i \in \{ H, L \} \). Substituting \( \lambda_{1i} \) and (A24) into (A18), and solving the two equations (given by (A18) for \( i \in \{ H, L \} \)) for \( g' \) and \( \lambda_{2H} \) in terms of \( \lambda_{2L} \), we obtain

\( A25 \) \[ g' = \frac{(\lambda_{2L} \theta_H + \theta_L) (1 + \delta - 2\delta p)}{1 - \delta p + \lambda_{2L} \delta (1 - p)}. \]
A25 \[
\lambda_{2H} = \frac{(\theta_H - \theta_L)(1 - \delta p) - \lambda_{2L}\theta_H(1 - \delta)}{\lambda_{2L}\delta(\theta_H - \theta_L)(1 - p) - \theta_L(1 - \delta)}.
\]

Consider the low-polarization case in which \(\frac{\theta_H}{\theta_L} \leq \frac{1 - \delta p}{\delta(1 - p)}\). Note that 
\(\delta(\theta_H - \theta_L)(1 - p) - \theta_L(1 - \delta) \leq 0\). Since \(\lambda_{2L} \leq 1\) by (A16) and (A17), it follows
that the denominator of (A26) is nonpositive. Together with the necessary condition that
\(\lambda_{2H} \geq 0\), this implies
\[
\lambda_{2L} \geq \frac{(\theta_H - \theta_L)(1 - \delta p)}{\theta_H(1 - \delta)}.
\]

Thus, if \(\frac{\theta_H}{\theta_L} \leq \frac{1 - \delta p}{\delta(1 - p)}\), we have \(\lambda_{2L} \in \left[\frac{(\theta_H - \theta_L)(1 - \delta p)}{\theta_H(1 - \delta)}, 1\right]\). Since the right-hand side of (A25) is increasing in \(\lambda_{2L}\), the bounds on \(\lambda_{2L}\) we just found implies that 
\(g = g' \in [\theta_H, \theta_H + \theta_L]\).

Next consider the high-polarization case in which \(\frac{\theta_H}{\theta_L} > \frac{1 - \delta p}{\delta(1 - p)}\). Note that 
\(\frac{(\theta_H - \theta_L)(1 - \delta p)}{\theta_H(1 - \delta)} > 1\). Since \(\lambda_{2H} \geq 0\), the numerator and the denominator of (A26) have the same sign. If they are both nonpositive, then
\[
\frac{(\theta_H - \theta_L)(1 - \delta p)}{\theta_H(1 - \delta)} \leq \lambda_{2L}.
\]

Since \(\lambda_{2L} \leq 1\) by (A16) and (A17), this is ruled out.

If instead both the numerator and the denominator of (A26) are nonnegative, then, since \(\lambda_{2H} \leq 1\) by (A16) and (A17), we have
\[
(\theta_H - \theta_L)(1 - \delta p) - \lambda_{2L}\theta_H(1 - \delta) \leq \lambda_{2L}\delta(\theta_H - \theta_L)(1 - p) - \theta_L(1 - \delta).
\]

Rearranging the expression above gives \(\theta_H(1 - \delta p) - \theta_L\delta(1 - p) \leq \lambda_{2L}[\theta_H(1 - \delta p) - \theta_L\delta(1 - p)]\). Since \(\theta_H \geq \theta_L\), the left-hand side is greater than zero. Dividing both sides by the expression on the left-hand side gives \(1 \leq \lambda_{2L}\). This is only possible if \(\lambda_{2L} = 1\) since \(\lambda_{2L} \leq 1\). Thus, in the high-polarization case, \(\lambda_{2L} = 1\). Substituting in (A25), we obtain \(g' = g = \theta_H + \theta_L\).

D. Proof of Proposition 7

If public good spending is discretionary, then party \(i\)’s expected steady state payoff is
\[
(A27) \quad \frac{1}{2(1 - \delta)} \left[\left(1 - \theta_i\right) + \theta_i \ln(\theta_i)\right] + \frac{1}{2(1 - \delta)} \left[\theta_i \ln(\theta_i)\right].
\]

If public good spending is mandatory, then party \(i\)’s expected steady state payoff is
\[
(A28) \quad \frac{1}{2(1 - \delta)} \left[\left(1 - g^i\right) + \theta_i \ln(g^i)\right] + \frac{1}{2(1 - \delta)} \left[\theta_i \ln(g^i)\right],
\]
where $g^s \in [g^s_H, \theta_H + \theta_L]$ in the low-polarization case and $g^s = \theta_H + \theta_L$ in the high-polarization case. To show that party $i$ is better off when public spending is mandatory, we only need to show that (A28) is higher than (A27). After rearranging terms, it becomes

\[(A29) \quad 2\theta_i \ln(g^s) - g^s \geq \theta_i \ln(\theta_i) - \theta_i.\]

Consider first $i = H$. Let $k(x) = 2\theta_H \ln(x) - x$. Since $k'(x) = \frac{2\theta_H}{x} - 1 > 0$ if $x < 2\theta_H$, and $g^s \leq \max\{g^s_H, \theta_H + \theta_L\} < 2\theta_H$, we have $k(g^s) > k(\theta_H)$. That is, $2\theta_H \ln(g^s) - g^s > 2\theta_H \ln(\theta_H) - \theta_H$. Since $\ln(\theta_H) > \ln(\theta_l\theta_H)$, it follows that $2\theta_H \ln(g^s) - g^s > \theta_H \ln(\theta_l\theta_H) - \theta_H$.

Next consider $i = L$ in the low-polarization case. Since the left-hand side of inequality (A29) is concave in $g^s$, we have (A29) holds for any $g^s \in [g^s_H, \theta_H + \theta_L]$ if it holds for $g^s = g^s_H$ and for $g^s = \theta_H + \theta_L$. If $g^s = g^s_H$, then

\[2\theta_L \ln(g^s) - g^s - \theta_L \ln(\theta_H\theta_L) + \theta_L = 2\theta_L \ln(g^s_H) - g^s_H - \theta_L \ln(\theta_H\theta_L) + \theta_L,\]

which is increasing in $\theta_L$. Let $\theta_L = \frac{\delta(1-p)}{1 - \delta p} \theta_H$. Then

\[2\theta_L \ln(g^s_H) - g^s_H - \theta_L \ln(\theta_H\theta_L) + \theta_L\]

\[= \ln \left( \frac{(1 + \delta - 2\delta p)^2}{\delta(1-p)(1 - \delta p)} \right) \frac{\delta(1-p)}{1 - \delta p} \theta_H - \theta_H,\]

and it is positive if $\ln \left( \frac{(1 + \delta - 2\delta p)^2}{\delta(1-p)(1 - \delta p)} \right) \geq \frac{1 - \delta p}{\delta(1-p)}$. Similarly, if $g^s = \theta_H + \theta_L$, then

\[2\theta_L \ln(g^s) - g^s - \theta_L \ln(\theta_H\theta_L) + \theta_L\]

\[= 2\theta_L \ln(\theta_H + \theta_L) - \theta_L - \theta_H - \theta_L \ln(\theta_H\theta_L) + \theta_L\]

\[= 2\theta_L \ln(\theta_H + \theta_L) - \theta_H - \theta_L \ln(\theta_H\theta_L),\]

which is increasing in $\theta_L$. Let $\theta_L = \frac{\delta(1-p)}{1 - \delta p} \theta_H$. Then

\[2\theta_L \ln(\theta_H + \theta_L) - \theta_H - \theta_L \ln(\theta_H\theta_L)\]

\[= \ln \left( \frac{(1 + \delta - 2\delta p)^2}{\delta(1-p)(1 - \delta p)} \right) \frac{\delta(1-p)}{1 - \delta p} \theta_H - \theta_H,\]

and it is positive if $\ln \left( \frac{(1 + \delta - 2\delta p)^2}{\delta(1-p)(1 - \delta p)} \right) \geq \frac{1 - \delta p}{\delta(1-p)}$. To summarize, inequality (A29) holds for $i = L$ if

\[\ln \left( \frac{(1 + \delta - 2\delta p)^2}{\delta(1-p)(1 - \delta p)} \right) \geq \frac{1 - \delta p}{\delta(1-p)}.\]
Let
\[
w(\delta, p) = \ln \left( \frac{(1 + \delta - 2\delta p)^2}{\delta(1 - p)(1 - \delta p)} \right) - \frac{1 - \delta p}{\delta(1 - p)}.
\]

Notice if \( \delta = 1 \), then \( w(\delta, p) = \ln(4) - 1 > 0 \). Hence, if the parties are sufficiently patient, then even the party who places a lower weight on public goods is better off ex ante if the spending on public goods is mandatory.

It is straightforward to verify that \( w(\delta, p) \) is decreasing in \( p \) and increasing in \( \delta \). When \( p = 0 \), \( w(\delta, p) = \ln \left( \frac{(1 + \delta)^2}{\delta} \right) - 1/\delta \), and \( w(\delta, 0) = 0 \) when \( \delta \approx 0.706 \). It follows that if \( \delta > 0.706 \), then there exists \( p > 0 \) such that for all \( p < p^* \), \( w(\delta, p) > 0 \), and even party \( L \) benefits ex ante from mandatory public good spending.

REFERENCES


\[\text{REFERENCE}\]


